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Dirac equation for the supermembrane in a background with fluxes from a component description of the $D = 11$ supergravity-supermembrane interacting system

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Abstract: We present a simple derivation of the 'Dirac' equation for the supermembrane fermionic field in a $D = 11$ supergravity background with fluxes by using a complete but gauge-fixed description of the supergravity-supermembrane interacting system previously developed. We also discuss the contributions linear in the supermembrane fermions — the Goldstone fields for the local supersymmetry spontaneously broken by the superbrane — to the field equations of the supergravity-supermembrane interacting system. The approach could also be applied to more complicated dynamical systems such as those involving the M5-brane and the $D = 10$ Dirichlet branes.

Keywords: Space-Time Symmetries, Supergravity Models, M-Theory, p-branes.

Contents

1. Introduction

There has been a growing interest in the fermionic equations for superbranes in a supergravity background with fluxes (see [1, 2] and [3−6] for earlier papers) as these are needed to study nonperturbative effects in string theory. To find such equations, one takes the super–p–brane action in curved superspace and expands it in powers of the fermionic coordinate function $\hat{\theta}^{\check{\beta}}(\xi)$ [2−6] or proceeds in the same manner directly from the superfield fermionic equation itself [1]. To make such a decomposition one uses the Wess-Zumino (WZ) gauge for a superfield supergravity background plus the superspace supergravity constraints. For $D = 10, 11$ these superspace constraints imply the 'free' supergravity equations of motion, without any contribution from the superbrane source. Hence the 'Dirac' equation for $\hat{\theta}(\xi)$, as derived in [1] from the superembedding approach [7] and in [2, 4 − 6] from the $\mathcal{O}(\hat{\theta}(\xi)^{\wedge 2})$ actions for M2 and various super–Dp-branes, apparently involves a gravitational and gauge field background that satisfies the 'free' supergravity equations without any contribution from the superbrane. Thus the consistency of the results obtained in this approach, although widely believed, is not manifest. A way to check this consistency would be to re-derive the same equations from a system of equations providing a fully dynamical description of the supergravity-super-p-brane interacting system by using a well defined approximation.

As, by definition, a super-p-brane is a p-brane moving in superspace, a complete system of equations, including those for the supergravity fields with contributions from the super-p-brane, could be derived from the sum of the superbrane action and a *superfield* action for supergravity. Such an action can be studied in lower dimensions (see [8 − 10]), but a superfield action for $D = 10, 11$ supergravity is unknown. This made difficult the study the $D = 11$ supergravity-M-brane and $D = 10$ type II supergravity-Dp-brane interacting systems, the most interesting ones in an M-theoretical perspective.

This difficulty may be overcome by using the gauge-fixed description [8 − 10] provided by the sum of the spacetime (component) action for supergravity (without auxiliary fields) and that for a *bosonic* brane, as given by the purely bosonic limit of the superbrane action. From the point of view of the superfield formulation of the interacting system (hypothetical for $D = 10, 11$) the gauge is provided by the conditions of the WZ gauge for the supergravity superfields plus the condition $\hat{\theta}^\alpha(\xi) = 0$ for the superbrane coordinate functions. The resulting gauge-fixed description is complete [9] in the sense that it contains gauge-fixed versions of all the dynamical equations of motion of the interacting system, including a 'fermionic equation for the bosonic brane' [11]. This equation, which formally coincides with the leading component of the superfield fermionic equation for the superbrane in a superspace supergravity background, appears in this component scheme as a selfconsistency condition for the bulk gravitino equations. Note that the 'fermionic equation for bosonic brane' is actually a non-dynamical 'boundary' condition for the gravitino on the brane worldvolume W^{p+1}. However, we will see how this algebraic equation allows us to obtain the superbrane fermionic field dynamics (the 'Dirac equation'), which is hidden in it.

The above gauge-fixed action for the supergravity-superbrane interacting system can be derived from the superfield description in the dimensions where a superfield supergravity action exists (see [8, 10] for $D = 4$, $N = 1$ interacting systems). In the general case its form may be also deduced if one assumes the existence of a superfield supergravity action and exploits its defining properties [9]. Then one concludes that, whether a superfield description of a supergravity-superbrane interacting system exists or not, the description of this system by means of the sum of the spacetime component supergravity action without auxiliary fields and the action of the 'limiting' bosonic brane (obtained by taking the purely bosonic, $\hat{\theta}^\alpha(\xi) = 0$, 'limit' of the supermembrane) does exist. Such an action preserves one-half of the local supersymmetry [11] characteristic of the pure supergravity action. This one-half of the local supersymmetry reflects the κ-symmetry of the original superbrane action while the existence of the non-preserved one-half reflects the spontaneous partial breaking of the local supersymmetry by the superbrane.

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In this paper we show that this complete but gauge-fixed description of the supergravity-superbrane interacting system may be used, despite it corresponds to the $\hat{\theta}^{\alpha}(\xi) = 0$ gauge for the superbrane variables, to reproduce the superbrane fermionic equation i.e., the dynamical Dirac equation for the fermionic field $\hat{\theta}^{\alpha}(\xi)$, in a supergravity background with fluxes. This is related to the known fact that the superbrane fermionic coordinate functions are the Goldstone fields for the supersymmetry spontaneously broken by the superbrane. We also discuss the possibility of using the Goldstone nature of the $\hat{\theta}^{\alpha}(\xi)$'s to find their lower order ($\mathcal{O}(\hat{\theta}^{\wedge k})$) contributions to the bulk supergravity equations, i.e. to search for a lower-order approximation in $\hat{\theta}^{\alpha}(\xi)$ to the system of interacting equations that would possess full local supersymmetry (not just the one half preserved by the gauge-fixed description of [9, 8, 10]) in the same (actually, $\mathcal{O}(\hat{\theta}^{\wedge(k-1)})$) approximation.

For definiteness we consider here the case of the $D = 11$ supergravity-supermembrane (*SG-M2*) interaction, although the method could be applied to other systems like the *SG-M5* one involving the M5-brane, or the *SG-Dp* system, with $D = 10$ type-II Dirichlet p-branes.

2. Supergravity interacting with a bosonic membrane as a gauge-fixed description of the $D = 11$ supergravity-supermembrane (*SG-M2*) interacting system

2.1 $D = 11$ Supermembrane in the on-shell superfield supergravity background

The supermembrane action in a supergravity background is [12]

$$S_{M2}[\hat{E}^a, \hat{A}_3] = \int_{W^3} \left(d^3\xi \frac{1}{2}\sqrt{|g(\xi)|} - \hat{A}_3 \right) = \int_{W^3} \left(\frac{1}{3!}\hat{E}^a \wedge \hat{\star}\hat{E}^a - A_3(\hat{Z}) \right), \tag{2.1}$$

where the pull-backs to the supermembrane worldvolume W^3

$$\hat{E}^a := d\hat{Z}^M(\xi) E_M{}^a(\hat{Z}(\xi)) =: d\xi^m \hat{E}^a_m$$

and

$$\hat{A}_3 = \frac{1}{3!} d\hat{Z}^{M_3} \wedge d\hat{Z}^{M_2} \wedge d\hat{Z}^{M_1} A_{M_1 M_2 M_3}(\hat{Z}(\xi)) \tag{2.2}$$

of the bosonic supervielbein $E^a = dZ^M E_M{}^a(Z)$ and the 3-superform $A_3 = \frac{1}{3!} dZ^{M_3} \wedge dZ^{M_2} \wedge dZ^{M_1} A_{M_1 M_2 M_3}(Z)$ of the superspace formulation [13, 14] of $D = 11$ supergravity [15] are denoted by a caret. These are obtained by replacing the superspace coordinate $Z^M = (x^\mu, \theta^{\check{\alpha}})$ by the coordinate functions $\hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), \hat{\theta}^{\check{\alpha}}(\xi))$ that 'locate' the worldvolume W^3 as a hypersurface in superspace, $Z^M = \hat{Z}^M(\xi)$. The worldvolume Hodge star operator $\hat{\star}$ in (2.1) is defined by

$$\hat{\star}\, \hat{E}^a = \frac{1}{2} d\xi^n \wedge d\xi^m \epsilon_{mnk} \sqrt{|g(\xi)|}\, g^{kl} \partial_l \hat{Z}^M\, E_M{}^a(\hat{Z})\,, \tag{2.3}$$

where $g_{mn}(\xi)$ is the induced metric on W^3,

$$g_{mn}(\xi) := \hat{E}^a_m\, \hat{E}_{n\,a} := \partial_m \hat{Z}^M \partial_n \hat{Z}^N\, E_N{}^a(\hat{Z})\, E_{M\,a}(\hat{Z})\,, \qquad |g(\xi)| := |\det(g_{mn}(\xi))|\,. \tag{2.4}$$

The supermembrane equations of motion

$$\hat{D}(\hat{\star}\hat{E}_a) = -\frac{1}{3}\hat{E}^d \wedge \hat{E}^c \wedge \hat{E}^b\, F_{abcd}(\hat{Z}) - \frac{1}{2}\hat{E}^d \wedge \hat{E}^\alpha \wedge \hat{E}^\beta \Gamma_{ab\,\alpha\beta}\,, \tag{2.5}$$

$$\hat{\star}\hat{E}^a \wedge \hat{E}^\beta\,(\Gamma_a(1-\bar{\bar{\gamma}}))_{\beta\alpha} = 0 \qquad \Leftrightarrow \qquad \hat{E}^\beta_m\left(\hat{\Gamma}^m(1-\bar{\bar{\gamma}})\right)_{\beta\alpha} = 0\,, \tag{2.6}$$

where D is the standard covariant derivative involving the spin connection, $\hat{E}^A_m := \partial_m \times \hat{Z}^M E^A_M(\hat{Z})$, $\hat{\Gamma}^m := g^{mn}(\xi)\hat{E}^a_n\,\Gamma_a$ and

$$\bar{\bar{\gamma}} = \frac{i}{3!\sqrt{|g(\xi)|}}\,\epsilon^{mnk}\partial_k\hat{Z}^K\partial_n\hat{Z}^N\partial_m\hat{Z}^M E^a_M(\hat{Z})E^b_N(\hat{Z})E^c_K(\hat{Z})\Gamma_{abc}\,, \tag{2.7}$$

are obtained [12] making use of the superspace supergravity constraints [13, 14]

$$T^a := DE^a := dE^a - E^b \wedge \omega_b{}^a = -iE^\alpha \wedge E^\beta\Gamma^a_{\alpha\beta}\,, \tag{2.8}$$

$$dA_3 = \frac{1}{4}E^\alpha \wedge E^\beta \wedge E^b \wedge E^a\Gamma_{ab\,\alpha\beta} + F_4(Z)\,, \quad F_4(Z) := \frac{1}{4!}E^{a_4} \wedge \cdots \wedge E^{a_1}F_{a_1\ldots a_4}(Z)\,. \tag{2.9}$$

These are known to be *on-shell* constraints i.e., they include the equations of motion for the physical spacetime or 'component' fields $e^a_\mu(x)$, $\psi^\alpha_\mu(x)$, $A_{\mu\nu\rho}(x)$,

$$e^a_\mu(x):\;\; e^a(x) := dx^\mu e_\mu{}^a(x) = E^a(Z)|_{\theta=0,\,d\theta=0} \tag{2.10}$$

$$\psi^\alpha_\mu(x):\;\; \psi^\alpha(x) := dx^\mu\psi^\alpha_\mu(x) = E^\alpha(Z)|_{\theta=0\,d\theta=0} \tag{2.11}$$

$$A_{\mu\nu\rho}(x):\;\; A_3(x) := 1/3!\,dx^\mu \wedge dx^\nu \wedge dx^\rho A_{\mu\nu\rho}(x) = A_3(Z)|_{\theta=0,\,d\theta=0}\,, \tag{2.12}$$

among their consequences. For our present discussion it is important to note that these are 'free' supergravity equations in the sense that they do not contain any source contribution from the supermembrane.

The on-shell supergravity constraints (2.8), (2.9) are also necessary conditions for the κ-symmetry of the supermembrane in a curved superspace background [12]. This local fermionic symmetry, first found in a superparticle context in [16], manifests itself by the presence of the projector $(1-\bar{\bar{\gamma}})$ in the fermionic equations (2.6). Its explicit form is given by

$$\delta_\kappa\hat{Z}^M(\xi) = (1+\bar{\bar{\gamma}})^\alpha{}_\beta\,\kappa^\beta(\xi)\,E_\alpha{}^M(\hat{Z})\,. \tag{2.13}$$

When the supergravity background superfields obey the Wess-Zumino (WZ) gauge conditions

$$\begin{aligned} i_\theta E^a &:= \theta^{\check{\beta}}E_{\check{\beta}}{}^a(x,\theta) = 0\,,\\ i_\theta E^\alpha &:= \theta^{\check{\beta}}E_{\check{\beta}}{}^\alpha(x,\theta) = \theta^{\check{\beta}}\delta_{\check{\beta}}{}^\alpha =: \theta^\alpha\,,\\ &\quad i_\theta A_3(Z) = 0\,, \qquad i_\theta\omega^{ab}(Z) = 0 \end{aligned} \tag{2.14}$$

(see [8] and refs. therein; see also [17])[1] the index of the Grassmann coordinate θ and of the coordinate function $\hat{\theta}(\xi)$ is identified with the spinor index and, furthermore, one can extract from (2.13) the κ-symmetry transformations for the $\hat{\theta}^\alpha$ variable,

$$\delta_\kappa\hat{\theta}^\alpha(\xi) = (1+\bar{\bar{\gamma}})^\alpha{}_\beta\,\kappa^\beta(\xi) + \mathcal{O}(\hat{\theta}) = (1+\bar{\gamma})^\alpha{}_\beta\,\kappa^\beta(\xi) + \mathcal{O}(\hat{\theta})\,, \tag{2.15}$$

where in eq. (2.15) $\bar{\gamma} = \bar{\bar{\gamma}}|_{\hat{\theta}=0}$.

[1] i_θ is a shorthand for the contraction with the vector field $\theta^{\check{\alpha}}\frac{\partial}{\partial\theta^{\check{\alpha}}}$.

The name *Dirac equation for the superbrane* is usually given [1, 2] for (an approximation to) the equation of motion for the superbrane fermion $\hat{\theta}(\xi)$ in a *spacetime* supergravity background. We will also call it below *superbrane fermionic equation*. In the standard approach to derive this equation [4−6, 1, 2] one considers the action (2.1) [4−6, 2] or [1] the superfield fermionic equation (2.6) for the on-shell supergravity background taken in the WZ gauge and expands it in powers of $\hat{\theta}(\xi)$ keeping the lower orders in $\hat{\theta}(\xi)$; the first order is usually considered to be sufficient. Then one uses the κ-symmetry (2.13) to gauge away half (16 out of 32) of the $\hat{\theta}(\xi)$ components to retain only the physical supermembrane fermions.

The fact that both the very derivation of the superfield fermionic equation (2.6) and its decomposition in powers of $\hat{\theta}(\xi)$ makes an essential use of the *on-shell* superspace supergravity constraints, which cannot incorporate any supermembrane source contribution, makes the consistency of the standard background superfield approach [4−6, 1, 2] not obvious. The check of its consistency is one of the motivations of this paper.

2.2 On the properties of a (hypothetical) superfield lagrangian description of the $D = 11$ supergravity-superbrane interaction

A complete, supersymmetric description of the *SG-M2* interaction would be provided by the sum

$$\mathbf{S}_{SG-M2} = \mathbf{S}_{SG}[E^a, E^\alpha, A_3(Z)] + S_{M2}[\hat{E}^a, A_3(\hat{Z})] \tag{2.16}$$

of the supermembrane action (2.1) and the hypothetical superfield action for $D = 11$ supergravity $\mathbf{S}_{SG}[E^a, E^\alpha, A_3(Z)]$. This action is not known and it is not even clear whether it exists. Nevertheless, if exists, such a supergravity action would possess certain properties. In particular, it would be invariant under arbitrary changes of the superspace coordinates, i.e. superdiffeomorphisms δ_{sdiff}. The same is true of the full interacting action (2.16) provided [8, 9] that the transformations of the *coordinate functions* of superbrane, $\hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), \hat{\theta}^{\check{\alpha}}(\xi))$ are given by the pull-backs $\hat{b}^M = b^M(\hat{Z}(\xi))$ to W^3 of the superspace diffeomorphism parameters $b^M(Z)$, i.e.

$$\delta_{sdiff} Z^M = b^M(Z)\,, \qquad \delta_{sdiff} \hat{Z}^M(\xi) = b^M(\hat{Z}(\xi))\,. \tag{2.17}$$

Eq. (2.17) implies, in particular,

$$\delta_{sdiff} \theta^{\check{\alpha}} = b^{\check{\alpha}}(Z)\,, \qquad \delta_{sdiff} \hat{\theta}^{\check{\alpha}}(\xi) = b^{\check{\alpha}}(\hat{Z}(\xi))\,. \tag{2.18}$$

Clearly, the transformations $\delta_{sdiff} Z^M = b^M(Z)$ cannot be used to set the fermionic *coordinates* θ^α equal to zero since such a transformation would have a vanishing superdeterminant and, hence, would not be a superdiffeomorphism. However, in contrast, the transformations (2.18) *can* be used to make the fermionic coordinate *functions* $\hat{\theta}^\alpha(\xi)$ vanishing, i.e. one can fix the gauge [8, 9]

$$\hat{\theta}^\alpha(\xi) = 0\,, \tag{2.19}$$

which might be considered the analogue to the 'unitary gauge' of the Higgs model.

Another expected property of the hypothetical superfield interacting action (2.16) is that, in addition to the superspace diffeomorphism gauge symmetry (eqs. (2.17), (2.18)), it

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would possess a local 16-parametric fermionic κ-symmetry δ_κ acting on the supermembrane variables $\hat{Z}^M(\xi)$ only. It is also plausible to assume that such a κ-symmetry would be characterized by eq. (2.13) with some superfield projector $1/2(1+\bar{\bar{\gamma}})$, $\bar{\bar{\gamma}} \equiv \bar{\bar{\gamma}}(Z)$. Thus the set of fermionic gauge symmetries of the action would contain $\delta_{\text{gauge}} = \delta_{sdiff} + \delta_\kappa$. These transformations act on $\hat{\theta}^\alpha(\xi)$ as

$$\delta_{\text{gauge}}\hat{\theta}^\alpha(\xi) = b^\alpha(\hat{Z}(\xi)) + \delta_\kappa\hat{\theta}^\alpha(\xi) = -\varepsilon^\alpha(\hat{x}) + \mathcal{O}(\hat{\theta}(\xi)) + \delta_\kappa\hat{\theta}^\alpha(\xi)\,, \tag{2.20}$$

where the leading component of the superfield superdiffeomorphism parameter has been denoted by $-\varepsilon^\alpha(\hat{x})$,

$$b^\alpha(\hat{Z}(\xi)) = -\varepsilon^\alpha(\hat{x}) + \mathcal{O}(\hat{\theta}(\xi)) \tag{2.21}$$

to identify $\varepsilon^\alpha(x)$ with the spacetime local supersymmetry parameter.

Irrespective of the details of the superspace formulation of supergravity, the WZ gauge (2.14) can be fixed on the supergravity superfields (see e.g. [13, 8] and refs. therein) by using superdiffeomorphism symmetry (2.17) and the superspace structure group symmetry, SO(1, 10) in the present case. The WZ gauge is then preserved by a certain combination of the superdiffeomorphism and the superspace local Lorentz group transformations expressed in terms of a number of independent parameters, $\epsilon^\alpha(x)$ of the spacetime local supersymmetry, $b^\mu(x)$ of spacetime diffeomorphisms and $L^{ab}(x)$ of spacetime local Lorentz transformations. In the WZ gauge the transformations of the fermionic coordinate function of the superbrane, $\hat{\theta}^\alpha(\xi)$, read

$$\delta_\kappa\hat{\theta}^\alpha(\xi) = (1+\bar{\bar{\gamma}}(\hat{Z}))^\alpha{}_\beta\kappa^\beta(\xi) = (1+\bar{\gamma}(\hat{x}))^\alpha{}_\beta\kappa^\beta(\xi) + \mathcal{O}(\hat{\theta}(\xi))\,, \tag{2.22}$$

where $\bar{\gamma} \equiv \bar{\gamma}(x)$ is the leading component of $\bar{\bar{\gamma}} \equiv \bar{\bar{\gamma}}(\hat{Z})$ in the κ-symmetry projector, $\bar{\bar{\gamma}}(\hat{Z})|_{\hat{\theta}=0} = \bar{\gamma}(x)$. Substituting (2.22) for $\delta_\kappa\hat{\theta}^\alpha(\xi)$ in (2.20) one finds

$$\delta_{\text{gauge}}\hat{\theta}^\alpha(\xi) = -\varepsilon^\alpha(\hat{x}) + (1+\bar{\gamma})^\alpha{}_\beta\kappa^\beta(\xi) + \mathcal{O}(\hat{\theta}(\xi))\,, \tag{2.23}$$

Eq. (2.23) exhibits, first of all, the Goldstone nature of the superbrane fermionic coordinate functions $\hat{\theta}^\alpha(\xi)$: $\hat{\theta}^\alpha(\xi)$ are the Goldstone fermions corresponding to the supersymmetry spontaneously broken by the superbrane (see [21, 8, 9] and refs. therein). In the supergravity-superbrane interacting system this supersymmetry is the *spacetime* local gauge symmetry which can be used to remove the Goldstone field by fixing the gauge (2.19). Secondly, eq. (2.23) makes transparent that the spontaneous breaking of the local supersymmetry by superbrane is partial. Indeed, the simple observation

$$\hat{\theta}^\alpha(\xi) = 0 \;\Rightarrow\; 0 = \delta_{\text{gauge}}\hat{\theta}^\alpha(\xi)|_{\hat{\theta}(\xi)=0} = -\varepsilon^\alpha(\hat{x}) + (1+\bar{\gamma})^\alpha{}_\beta\kappa^\beta(\xi) \tag{2.24}$$

implies that the gauge (2.19) is preserved by a local supersymmetry of parameter $\varepsilon(x)$ whose pull-back to the brane is restricted by being of the form

$$\hat{\varepsilon}^\alpha(\xi) := \varepsilon^\alpha(\hat{x}) = (1+\bar{\gamma}(\hat{x}))_\beta\kappa^\beta(\xi)\,. \tag{2.25}$$

2.3 Gauge-fixed description of the *SG-M2* interacting system and its properties

Hence, as shown in [8, 9] and discussed above, in a hypothetical superfield description (2.16) of the supergravity-superbrane interacting system the gauge

$$\hat{\theta}^{\alpha}(\xi) = 0 \tag{2.26}$$

(eq. (2.19)) and the WZ gauge (2.14) may be fixed simultaneously. In the WZ gauge the integration over the Grassmann superspace coordinates θ^{α} in such a superfield action $\mathbf{S}_{SG}[E^a, E^{\alpha}, A_3(Z)]$ would produce a component spacetime supergravity action involving a (hypothetical) set of auxiliary fields. By definition, these auxiliary fields would satisfy algebraic equations which, used in the supergravity action, would lead to the standard supergravity action (in our case that of [15]) involving only the physical fields of the supergravity multiplet. This action is invariant under the local supersymmetry the algebra of which closes on-shell.

Notice that the auxiliary fields would be contained in the higher order components of $E_M^A(Z)$, $A_{MNK}(Z)$ (and, perhaps, in some additional auxiliary superfields). The leading ($\theta = 0$) components of the $E_M^A(Z)$ and $A_{MNK}(Z)$ superfields in the WZ gauge are either zero, unity, or, in the case of $E_\mu^A(Z)$ and $A_{\mu\nu\rho}(Z)$, determine the physical fields $e_\mu^a(x)$, $\psi^{\alpha}(x)$ and $A_{\mu\nu\rho}(Z)$ of the Cremmer-Julia-Scherk (CJS) supergravity multiplet [15] (eqs. (2.10), (2.11) and (2.12)). As a result, in the gauge defined by $\hat{\theta}(\xi) = 0$ (eq. (2.26)) plus the WZ gauge (eqs. (2.14)), the supermembrane action (2.1) reduces to the action S^0_{M2} of a purely *bosonic membrane* coupled to the *physical bosonic fields* of the supergravity multiplet only; neither the gravitino nor the auxiliary fields enter the membrane part of the gauge-fixed interacting action.

Hence, in the supergravity part of such a gauge-fixed action for the *SG-M2* interacting system one may remove the auxiliary fields through their algebraic equations in the same manner that one would do for the (also hypothetical) pure supergravity action with auxiliary fields. As a result one would arrive at the following gauge-fixed action for the *SG-M2* interacting system [9, 8] (see also [11])

$$S^0_{SG-M2} = S_{SG}[e^a, \psi^{\alpha}, A_3] + S^0_{M2} = \int_{M^{11}} \mathcal{L}_{11}[e^a, \psi^{\alpha}, A_3] + \int_{W^3} \left(\frac{1}{3!} \hat{e}^a \wedge \hat{*}\hat{e}_a - A_3(\hat{x}) \right), \tag{2.27}$$

where $S_{SG} = S_{SG}[e^a, \psi^{\alpha}, A_3]$ is the standard CJS action for D=11 supergravity [15] and the second term is the action for a purely bosonic brane where the relative coefficient between its two terms is fixed (for a given supergravity action $S_{SG}[e^a, \psi^{\alpha}, A_3]$ invariant under definite supersymmetry transformations) since $S^0_{M2} = S_{M2}[e^a(\hat{x}), A_3(\hat{x})]$ is the bosonic limit of the M2-superbrane action $S_{M2}[E^a(\hat{Z}), A_3(\hat{Z})]$ [12] of eq. (2.1).

The following properties [11, 9] of the gauge-fixed action (2.27) will be important

1. The gauge-fixed description (2.27) of the supergravity-superbrane interacting system (2.16) is complete in the sense that it produces a gauge-fixed version of all the dynamical equations that would be obtained from a possible superfield action, including the *'fermionic equation for bosonic brane'* [11, 8], which is given by an algebraic

condition on the pull-back $\hat{\psi}^\alpha := d\xi^m \hat{\psi}_m^\alpha(\xi)$ of the gravitino to W^3. It states that a projection of a gamma-trace of $\hat{\psi}_m^\alpha$ vanishes, i.e. that

$$\hat{\psi}_m \hat{\Gamma}^m (1-\bar{\gamma}) = 0 \tag{2.28}$$

(see section 2.3.2), where

$$\hat{\psi}_m^\beta := \hat{e}_m^a(\xi)\psi_a^\beta(\hat{x}(\xi)) = \partial_m \hat{x}^\mu \psi_\mu^\alpha(\hat{x}) \,, \qquad \hat{e}_m^a := \partial_m \hat{x}^\mu(\xi) e_\mu{}^a(\hat{x}(\xi)) \,,$$
$$\hat{\Gamma}^n_{\alpha\beta} := g^{nm}(\xi)\hat{e}_m^a(\xi)\Gamma_{a\,\alpha\beta} \,, \tag{2.29}$$

$g^{nm}(\xi)$ is inverse of the induced metric $g_{mn}(\xi) = \hat{e}_m{}^a \hat{e}_{n\,a}$ (eq. (2.4) with $\hat{\theta}(\xi) = 0$) and

$$\bar{\gamma} := \frac{i}{3!\sqrt{|g(\xi)|}} \epsilon^{mnk} \hat{e}_m{}^a \hat{e}_n{}^b \hat{e}_k{}^c \Gamma_{abc} \tag{2.30}$$

(*cf.* eq. (2.7) for $\hat{\theta}(\xi) = 0$) has the properties $\bar{\gamma}^2 = I$, $tr(\bar{\gamma}) = 0$.

2. The equations of motion for the bosonic supergravity fields get (or may get) a source term contribution from the superbrane, while the gauge-fixed equations for the bulk fermionic fields are sourceless (see section2.3.1).

3. The action (2.27) possesses half of the local supersymmetries of the pure supergravity action $S_{SG}[e^a, \psi^\alpha, A_3]$. This is characterized [11] by the standard transformation rules for the supergravity fields [15] (see eqs. (2.47), (2.48), (2.49) below) and by the following conditions (see section 2.3.3) restricting the local supersymmetry parameter on the worldvolume W^3,

$$\hat{\varepsilon}^\alpha := \hat{\varepsilon}^\alpha(\hat{x}(\xi)) = (1+\bar{\gamma})^\alpha{}_\beta \kappa^\beta(\xi) \,. \tag{2.31}$$

4. The local supersymmetry algebra closes on-shell in exactly the same manner as it does for the case of free supergravity (section 2.3.4).

Property (1) might seem strange since no worldvolume fermionic degrees of freedom are seen directly in the gauge-fixed interacting action (2.27) involving the *bosonic* brane action. But this *'fermionic equation for the bosonic brane'* can be derived [11, 8] from the selfconsistency condition $\mathcal{D}\Psi_{10\,\alpha} = 0$ for the gravitino equation $\Psi_{10\,\alpha} = 0$ (eq. (2.35) below) which, according to (2), remains sourceless in the presence of the bosonic brane [11]. Thus, it is convenient to discuss property (2) first.

2.3.1 Field equations for the *SG-M2* system (property 2)

Varying the CJS action with respect to differentail forms $\delta S_{SG} = -2i\int \Psi_{10\,\alpha} \wedge \delta\psi^\alpha + \int \mathcal{G}_8 \wedge \delta A_3 + \int M_{10\,a} \wedge \delta E^a$ one can write the 'free' supergravity equations in differential form notation (see [18] and, e.g. [19]). The same can be done for the (gauge-fixed) field equations of the *SG-M2* interacting system, $\delta(S_{SG} + S^0_{M2}) = -2i\int \Psi_{10\,\alpha} \wedge \delta\psi^\alpha + \int(\mathcal{G}_8 - J_8) \wedge \delta A_3 + \int (M_{10\,a} - J_{10\,a}) \wedge \delta E^a$. The variation of the bosonic membrane part S^0_{M2} in the action is written as an integral over spacetime M^{11} with the use of the currents (see [11, 8]; $\hat{e}^b := d\xi^n \hat{e}_n{}^b(\xi)$, $\hat{e}_n{}^b := \partial_n \hat{x}^\mu(\xi) e_\mu{}^b(\hat{x})$)

$$
\begin{aligned}
J_{10\,a} &= \frac{1}{2e(x)} e_b^{\wedge 10} \int_{W^3} *\hat{e}_a \wedge \hat{e}^b \delta^{11}(x - \hat{x}(\xi)) \\
&= e_b^{\wedge 10} \int_{W^3} d^3\xi \, \frac{\sqrt{|g|}\, g^{mn} \hat{e}_{na} \hat{e}^b_m}{2|\det(e_\nu{}^c(\hat{x}))|} \, \delta^{11}(x - \hat{x}(\xi)) \,, \qquad (2.32) \\
J_8 &= \frac{1}{e(x)} e^{\wedge 8}_{abc} \int_{W^3} \hat{e}^a \wedge \hat{e}^b \wedge \hat{e}^c \, \delta^{11}(x - \hat{x}(\xi)) \\
&= e^{\wedge 8}_{abc} \int_{W^3} d^3\xi \frac{\epsilon^{mnk} \hat{e}^a_m \, \hat{e}^b_n \, \hat{e}^c_k}{|\det(e_\nu{}^c(\hat{x}))|} \, \delta^{11}(x - \hat{x}(\xi)) \,, \qquad (2.33)
\end{aligned}
$$

which describe the brane source terms in the Einstein and gauge field equations. The Einstein and the Rarita-Schwinger equations of the interacting system are written in terms of the ten-forms

$$
\begin{aligned}
M_{10\,a} &:= \frac{1}{4} R^{bc} \wedge e^{\wedge 8}_{abc} + \frac{1}{2} \left(i_a F_4 \wedge *F_4 + F_4 \wedge i_a(*F_4) \right) + \mathcal{O}(\psi^{\wedge 2}) + \mathcal{O}(\psi^{\wedge 4}) \\
&= J_{10\,a} \,, \qquad (2.34) \\
\Psi_{10\,\alpha} &:= \mathcal{D}\psi^\beta \wedge \bar{\Gamma}^{(8)}_{\beta\alpha} = 0 \,, \qquad (2.35)
\end{aligned}
$$

while the eight-form expression of the three-form gauge field equation reads

$$
\mathcal{G}_8 := d(*F_4 + b_7 - A_3 \wedge dA_3) = J_8 \,, \qquad b_7 := \frac{i}{2} \psi^\alpha \wedge \psi^\beta \wedge \bar{\Gamma}^{(5)}_{\alpha\beta} \,. \qquad (2.36)
$$

In eqs. (2.34), (2.35) and (2.36) the eight-forms $e^{\wedge 8}_{abc}$, $\bar{\Gamma}^{(8)}_{\beta\alpha}$ and the five form $\bar{\Gamma}^{(5)}_{\beta\alpha}$ are defined by

$$
e^{\wedge(11-q)}_{a_1 \dots a_q} := \frac{1}{(11-q)!} \epsilon_{a_1 \dots a_q b_1 \dots b_{11-q}} e^{b_1} \wedge \dots \wedge e^{b_{11-q}} \,, \qquad \bar{\Gamma}^{(q)} := \frac{1}{q!} e^{a_q} \wedge \dots \wedge e^{a_1} \Gamma_{a_1 \dots a_q} \,, \qquad (2.37)
$$

the four-form F_4 is the 'supersymmetric' field strength of the three-form gauge field A_3,

$$
F_4 := \tfrac{1}{4!} e^{a_4} \wedge \ldots \wedge e^{a_1} F_{a_1 \ldots a_4}(x) = dA_3 - \tfrac{1}{2} \psi^\alpha \wedge \psi^\beta \wedge \bar{\Gamma}^{(2)}_{\alpha\beta} \,, \qquad (2.38)
$$

and $*F_4 := -e^{\wedge 7}_{abcd} F^{abcd}$. The spin connection $\omega^{ab} = -\omega^{ba}$ are expressed through the graviton and gravitino by the solution of the torsion constraint

$$
T^a(x) := De^a = de^a - e^b \wedge \omega_b{}^a = -i\psi \wedge \Gamma^a \psi \,, \qquad (2.39)
$$

which formally coincides with the leading component of the on-shell superspace constraint (2.8). The explicit expressions for the two-fermionic and four-fermionic contributions, $\mathcal{O}(\psi^{\wedge 2})$ and $\mathcal{O}(\psi^{\wedge 4})$, to the Einstein equations (2.34) will not be needed in this paper. The generalized covariant derivative $\mathcal{D}\psi^\alpha$ in (2.35) is defined by

$$
\mathcal{D}\psi^\alpha = d\psi^\alpha - \psi^\beta \wedge \omega_\beta{}^\alpha \,, \quad \omega_\alpha{}^\beta = \frac{1}{4} \omega^{ab} \Gamma_{ab\alpha}{}^\beta + e^a t_{a\alpha}{}^\beta \qquad (2.40)
$$

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and contains, in addition to the spin connection $\frac{1}{4}\omega^{ab}\Gamma_{ab\alpha}{}^{\beta}$, the covariant contribution $e^a t_{a\alpha}{}^{\beta}$,

$$t_{a\,\beta}{}^{\alpha} := \frac{i}{18}\left(F_{ac_1c_2c_3}\Gamma^{c_1c_2c_3}{}_{\beta}{}^{\alpha} - \frac{1}{8}\,F^{c_1c_2c_3c_4}\Gamma_{ac_1c_2c_3c_4\,\beta}{}^{\alpha}\right), \qquad (2.41)$$

expressed through the 'supersymmetric' field strength $F_{abcd}(x)$ of A_3, eq. (2.38). This covariant part of the generalized connection thus describes the coupling of the bulk gravitino field to the fluxes of the three-form gauge field A_3.

The reason for the absence of source in the fermionic equation (2.35) obtained by varying the gauge-fixed action (2.27) with respect to the gravitino field is, clearly, that the bosonic brane action S^0_{M2} in (2.27) does not include the gravitino $\hat{\psi}^\alpha$; this, in turn, follows from the absence of the fermionic supervielbein $(E^\alpha(\hat{Z}))$ in the supermembrane action $\mathbf{S}_{M2}$ of eq. (2.1). Nevertheless, the absence of an explicit source term in (2.35) does not imply that the gravitino is decoupled from the brane source since eq. (2.35) includes the vielbein $e^a_\mu(x)$ (entering also through the composite spin connection ω^{ab}) and the field strength of the three-form gauge field A_3 that do obey the sourceful eqs. (2.34) and (2.36).

Notice that the system of interacting equations, including eqs. (2.34), (2.36), (2.35), (2.28) as well as the bosonic equation for the brane, admits particular solutions with $\psi^\alpha(x) = 0$. Inserting $\psi^\alpha(x) = 0$ back into the equations one arrives at the well-known system of purely bosonic supergravity equations in [20].

2.3.2 'Fermionic equations' for the bosonic brane interacting with supergravity (property 1)

To understand how the 'fermionic equation for the bosonic brane' results from the consistency conditions of the gravitino equation one can use the identity (see e.g. [19])

$$\mathcal{D}\Psi_{10\,\alpha} = i\psi^\beta \wedge \left(M_{10\,a}\Gamma^a_{\beta\alpha} + \frac{i}{2}\mathcal{G}_8 \wedge \bar{\Gamma}^{(2)}_{\beta\alpha}\right) \qquad (2.42)$$

that expresses the generalized covariant derivative of the *l.h.s.* of the fermionic equation (2.35) in terms of the left hand sides of the Einstein and the A_3 gauge field equations, $M_{10\,a}$ and $\mathcal{G}_8$ in eqs. (2.34) and (2.36), respectively. For 'free' $D = 11$ supergravity the equations of motion are $\Psi_\alpha = 0$, $M_{10\,a} = 0$ and $\mathcal{G}_8 = 0$, and the eleven-form *identity* (2.42) shows their interdependence. This *Noether identity* reflects a local gauge symmetry of the CJS supergravity action $S_{SG}[e^a, \psi^\alpha, A_3]$, the local supersymmetry of $D = 11$ supergravity [15] (eqs. (2.47)–(2.49) below).

When supergravity interacts with a bosonic membrane, $S_{SG} \mapsto S_{SG} + S^0_{M2}$ like in the gauge fixed description, the bosonic field equations acquire source terms and read $M_{10\,a} = J_{10\,a}$ [eqs. (2.34), (2.32)] and $\mathcal{G}_8 = J_8$ [eqs. (2.36), (2.33)]; the fermionic equation, however, remains sourceless, $\Psi_{10\alpha} = 0$ [eq. (2.35)]. Hence, eq. (2.42) produces the following equation for the M2-brane currents $J_{10\,a}$ and J_8 (see eqs. (2.32) and (2.33))

$$i\psi^\beta \wedge \left(J_{10\,a}\Gamma^a_{\beta\alpha} + \frac{i}{2}J_8 \wedge \bar{\Gamma}^{(2)}_{\beta\alpha}\right) = 0\,. \qquad (2.43)$$

Due to the currents, this eleven-form equation has support on the M2-brane worldvolume W^3 and so it can be written as a three-form equation on W^3 in terms of the pull-backs of the graviton and gravitino [11, 8]. When $S_{SG}+S^0_{M2}$ provides the gauge fixed description of the supergravity-supermembrane interaction the currents $J_{10\,a}$ and J_8 are defined by eqs. (2.32) and (2.33) and the equivalent form of eq. (2.43) reads

$$\hat{\psi}^\beta \wedge \left(i\,\hat{*}\hat{e}^a\,\Gamma_{a\,\beta\alpha} + \frac{1}{2}\hat{e}^b \wedge \hat{e}^c\,\Gamma_{bc\,\beta\alpha} \right) = 0\,. \tag{2.44}$$

A simple algebra allows us to present eq. (2.44) in the form of

$$\hat{\Xi}_{3\,\alpha} := \hat{*}\hat{e}^a \wedge \hat{\psi}^\beta\,(\Gamma_a(1-\bar{\gamma}))_{\beta\alpha} = 0 \tag{2.45}$$

where the action of $\hat{*}$ is defined by $\hat{*}\hat{e}^a = \frac{1}{2}d\xi^n \wedge d\xi^m \epsilon_{mnk}\sqrt{|g(\xi)|}g^{kl}\partial_l\hat{x}^\mu e_\mu{}^a(\hat{x})$ (eq. (2.3) with $\hat{\theta}=0$, i.e. for $\hat{*} = \hat{\star}|_{\hat{\theta}=0}$). Eq. (2.45) is an equivalent form of eq. (2.28) written in a conventional differential form notation,

$$\hat{\Xi}_{3\,\alpha} \propto d^3\xi\,\hat{\psi}_m\hat{\Gamma}^m(1-\bar{\gamma})\,. \tag{2.46}$$

2.3.3 Supersymmetry of the gauge-fixed action (property 3)

Eq. (2.42) is the Noether identity for the local supersymmetry of the pure supergravity action $S_{SG}[e^a, \psi^\alpha, A_3]$,

$$\delta_\varepsilon e^a = -2i\psi\Gamma^a\varepsilon := -2i\psi^\alpha\Gamma^a_{\alpha\beta}\varepsilon^\beta\,, \tag{2.47}$$

$$\delta_\varepsilon \psi^\alpha = \mathcal{D}\varepsilon^\alpha(x) = D\varepsilon^\alpha(x) - \varepsilon^\beta(x)e^a t_{a\beta}{}^\alpha(x)\,, \tag{2.48}$$

$$\delta_\varepsilon A_3 = \psi^\alpha \wedge \bar{\Gamma}^{(2)}_{\alpha\beta}\varepsilon^\beta\,, \tag{2.49}$$

where $\mathcal{D}$ is the generalized covariant derivative of eq. (2.40), $D = d - \omega$ is the standard covariant derivative, and the tensorial part of the generalized connection $t_{a\beta}{}^\alpha(x)$ is defined in eq. (2.41). The fact that eq. (2.42) is not identically satisfied in the presence of a bosonic brane, i.e. when $S_{SG} \mapsto S_{SG} + S^0_{M2}$, reflects the fact that the bosonic brane action S^0_{M2} breaks the local supersymmetry (2.47)–(2.49).

When the bosonic brane is the purely bosonic ($\hat{\theta}=0$) 'limit' of a superbrane, the sum of the supergravity action and the action of bosonic brane provides a gauge-fixed description of the supergravity — *super*brane (*SG-M2*) interacting system [8, 9] and preserves one-half of the local supersymmetry of S_{SG}. This half of the local supersymmetry is defined by the restriction (2.31) on the pull-back of the supersymmetry parameter to the membrane worldvolume W^3, $\hat{\varepsilon}^\alpha := \hat{\varepsilon}^\alpha(\hat{x}(\xi)) = (1+\bar{\gamma})^\alpha{}_\beta\kappa^\beta(\xi)$. Its preservation can be shown in two ways, either explicitly [11] (see also section 3.3 below) or using the fact that the action (2.27) provides a gauge-fixed version of the hypothetical superfield description of the supergravity-superbrane interaction [9, 8] as discussed also in section 2.2.

2.3.4 On-shell closure of the local supersymmetry algebra in the spacetime gauge-fixed description of *SG-M2* system (property 4)

As known from the pioneering paper [15], the local supersymmetry transformations (2.47)–(2.49) that leave invariant the supergravity action $S_{SG}[e^a, \psi^\alpha, A_3(x)]$ form an algebra which

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is closed on shell, i.e. using the 'free' supergravity equations. The structure of this algebra is schematically [15]

$$[\delta_{\epsilon_1}\,,\,\delta_{\epsilon_1}][\text{fields}] = \Big(\delta_{b^\mu(\epsilon_1,\,\epsilon_2)} + \delta_{L^{ab}(\epsilon_1,\,\epsilon_2)} + \delta_{\epsilon_3(\epsilon_1,\,\epsilon_2)} + \mathcal{K}_{(\epsilon_1,\,\epsilon_2)}\Big)\,[\text{fields}]\,, \qquad (2.50)$$

where δ_{b^μ} determines the general coordinate transformations, $\delta_{L^{ab}}$ the local Lorentz transformations, δ_ϵ the local supersymmetry transformations (2.47)–(2.49) and $\mathcal{K}_{(\epsilon_1,\,\epsilon_2)}[fields]$ denotes terms that express the non-closure of the algebra and that become zero on shell.

Let us now consider the *SG-M2* interacting system. The form of the supersymmetry transformations leaving invariant the coupled supergravity-bosonic brane action (2.27) (i.e. preserving the gauge $\hat{\theta}(\xi) = 0$ for the interacting supergravity-superbrane system [9, 8, 10]) is exactly the same as that of the supersymmetry of 'free' supergravity.[2] However, in principle, the last term in (2.50) might spoil the closure of the local supersymmetry algebra of supergravity-bosonic membrane system that provides the gauge-fixed description of the *SG-M2* interacting system). This is not so, however. On the bosonic fields of the supergravity multiplet, $e_\mu{}^a(x)$ and $A_{\mu\nu\rho}(x)$, the algebra (2.50) is closed off shell [15], i.e. without any use of the equations of motion. This means that

$$\mathcal{K}_{(\epsilon_1,\,\epsilon_2)}[e^a(x)] \equiv 0\,, \qquad \mathcal{K}_{(\epsilon_1,\,\epsilon_2)}[A_3(x)] \equiv 0\,. \qquad (2.51)$$

Hence the on-shell character of the supersymmetry algebra comes from the fermionic fields since only $\mathcal{K}_{(\epsilon_1,\,\epsilon_2)}[\psi] \neq 0$ off-shell.[3] Moreover, and this is the key point, only the fermionic equations are necessary to close supersymmetry algebra on the fermionic fields; schematically,

$$\mathcal{K}_{(\epsilon_1,\,\epsilon_2)}[\psi^\alpha(x)] \;\propto\; *\Psi_{10\alpha}\,. \qquad (2.52)$$

But as noticed above (following [11, 9, 8]), the fermionic equation for the interacting system in the gauge-fixed description given by the sum of supergravity action and the action for bosonic brane preserving a half of the local supersymmetry remains formally the same (i.e., sourceless) as that for 'free' supergravity, $\Psi_{10\alpha} = 0$. Hence,

$$\mathcal{K}_{(\epsilon_1,\,\epsilon_2)}[\psi^\alpha(x)]\big|_{\substack{\text{on-shell for the}\\ \text{interacting system}}} = 0\,. \qquad (2.53)$$

[2] Notice that this is not the case for the supersymmetric brane world models in [28]. There, the *brane* actions also contain the pull-back of the gravitino field. Probably these two facts are related and prevent or hamper a superfield formulation of the brane actions of [28]. In all other respects the models of [28] are similar to the dynamical systems of supergravity interacting with standard superbranes as they are presented in the gauge-fixed description of [9, 11] and section 2 of this paper. The breaking of 1/2 of the supersymmetry in the gauge-fixed description corresponds to imposing a kind of boundary conditions on the supersymmetry parameter in [28].

[3] The statement that in the absence of the auxiliary fields $\mathcal{K}_{(\epsilon_1,\,\epsilon_2)}[fermionic\,fields] \neq 0$ off-shell while $\mathcal{K}_{(\epsilon_1,\,\epsilon_2)}[bosonic\,fields] = 0$ seems to be quite general, i.e. valid for many supersymmetric theories in various dimensions, see e.g. [24]. To our knowledge, the only exception is provided by the supersymmetry transformations that preserve the equations of motion for supermultiplets that include self-dual gauge fields, where the selfduality condition for the bosonic gauge field is also needed to close the supersymmetry algebra.

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Further, the local supersymmetry transformations act only on the fields of supergravity multiplet, eqs. (2.47)–(2.49), since the only supermembrane field in the gauge-fixed description, the bosonic $\hat{x}(\xi)$, is inert under the local spacetime supersymmetry. Thus the on-shell closure of the local supersymmetry algebra of the gauge-fixed description of the supergravity-supermembrane interacting system follows from that of the pure $D = 11$ supergravity theory.

3. Goldstone nature of the supermembrane fermionic fields and Dirac equation for the supermembrane in a $D = 11$ supergravity background with fluxes

The Goldstone nature of the superbrane coordinate functions, in particular of the fermionic functions $\hat{\theta}^\alpha(\xi)$, has been known for a long time [21, 12]. For a superbrane interacting with *dynamical supergravity* the $\hat{\theta}^\alpha(\xi)$ are Goldstone (or compensator) fields for the local supersymmetry, a fact that explains the possibility of taking the gauge (2.26), $\hat{\theta}(\xi) = 0$, by using this local supersymmetry (see section 2.2).

In this gauge the lagrangian description of the system is provided by the sum (2.27) of the spacetime supergravity action without auxiliary fields and of the bosonic M2-brane action [9, 8]. The full set of equations of motion is given by the supergravity field equations (2.34), (2.36), (2.35), the bosonic brane equations (*cf.* eq. (2.5))

$$D(\hat{*}\hat{e}_a) = -2i_a F_4 - \frac{1}{2}\hat{e}^b \wedge \hat{\psi} \wedge \Gamma_{ab}\hat{\psi}\,, \qquad i_a F_4 := \frac{1}{3!}\hat{e}^d \wedge \hat{e}^c \wedge \hat{e}^b\, F_{abcd}(\hat{x})\,, \tag{3.1}$$

and the 'fermionic equation for the bosonic brane' [11], eq. (2.45), (*cf.* (2.6))

$$\hat{\Xi}_{3\,\alpha} = *\hat{e}^a \wedge \hat{\psi}^\beta\, (\Gamma_a(1-\bar{\gamma}))_{\beta\alpha} = 0\,. \tag{3.2}$$

In the $\hat{\theta}(\xi) = 0$ gauge, the fermionic degrees of freedom of the superbrane, usually associated with $\hat{\theta}(\xi)$, are contained in the pull-back $\hat{\psi}^\beta$ of the bulk gravitino to the worldvolume W^3 as *zero modes* corresponding to the supersymmetry broken by the brane.[4] Namely, the fact that half of the supersymmetry is spontaneously broken by the presence of the supermembrane is reflected by the explicit breaking of half of the local supersymmetry by the bosonic brane in the gauge-fixed description (2.27). Hence, in the presence of the supermembrane, the remaining local supersymmetry does not produce the same number of gauge-fixing conditions on the gravitino field as the full local supersymmetry of 'free' supergravity. Nevertheless, as shown in [11, 9], this super-Higgs effect in the presence of a superbrane does not make the gravitino massive, because the 'fermionic equation for the bosonic brane', eq. (2.45), takes the rôle of the lost gauge-fixing conditions and keeps the number of polarizations of the gravitino equal to those in 'free' supergravity. However, the fermionic zero modes corresponding to the supersymmetry broken by the membrane remain

[4]See [29] for a discussion of the brane degrees of freedom as zero modes, but starting from certain brane solutions of the supergravity equations.

in the pull-back $\hat{\psi}$ of the bulk gravitino ψ to W^3.[5] Precisely these zero modes represent the 16 fermionic degrees of freedom of supermembrane in the gauge-fixed description of (2.27).

To summarize, in the gauge-fixed description of the supergravity-supermembrane interaction provided by the set of equations (2.34), (2.36), (2.35), (3.1), (3.2), the bulk gravitino carries both the supergravity and the superbrane fermionic degrees of freedom as determined by the solution of field equation (2.35) with the boundary conditions (3.2) on the 3-dimensional 'defect', the brane worldvolume W^3. This description is convenient in studying the cases where both the effects from the bulk and from the worldvolume fermions are equally important and there is no need to separate their contribution.

However, in some cases (interesting e.g. for M-theory-based 'realistic' model building, see [4−6, 1, 2]) it may happen that the effects from the worldvolume fermions, and in particular the explicit form of their interaction with the flux, constitute the main interest. Then, when starting from our gauge-fixed description, one faces the problem of visualizing the fermionic degrees of freedom of the superbrane, i.e. the supermembrane coordinate functions $\hat{\theta}(\xi)$. This will be the main subject of the study below.

In the light of Goldstone nature of $\hat{\theta}(\xi)$, the general answer should not be too surprising: the recovery of the $\hat{\theta}(\xi)$ contributions to the action and equations of motion can be done by making (consistently) a local supersymmetry transformation the parameter of which is identified with the Goldstone fermion field $\hat{\theta}(\xi)$. We begin by showing how the supermembrane *fermionic equations in a supergravity background with fluxes*, this is to say with nonvanishing F_{abcd}, can be obtained on this way.

3.1 Dirac equation for the supermembrane in a supergravity background with fluxes from the gauge-fixed approach

When supergravity is treated as a background, one concentrates on the supermembrane equations. In our gauge-fixed description these are given by the bosonic equation (3.1) and the fermionic eq. (3.2) which is more a condition on the pull-back of the gravitino than a dynamical equation. To separate the contribution form the bulk fermions and from the

[5]To illustrate the above statement one can consider the weak field case, making a linear approximation in the fields to find the general solution of the equations; schematically, $\phi = \phi_0 + \sum(b(p)e^{+ip\cdot x} + b^\dagger(p)e^{-ip\cdot x})$. Then the number of polarizations (which distinguishes between the massive and massless cases) is determined by the oscillating contributions, and depends on the number of conditions imposed on the creation and annihilation operators $b^\dagger(p)$ and $b(p)$, while the zero modes are associated with the non-oscillating contribution ϕ_0.

The same occurs in the spacetime Higgs effect in general relativity interacting with branes [30, 9]. A bulk graviton does not get mass and keeps the same number of polarizations in the presence of a p-brane because the bosonic equations of the brane replace the gauge fixing conditions that were lost due to the spontaneous breaking of the diffeomorphism symmetry by the p-brane [11, 9, 30]. However, the corresponding gauge-fixing conditions for the case of *free* gravity allow for a residual gauge symmetry, which is absent when the bosonic equations of the brane take the rôle of the gauge fixing conditions. This set of residual gauge symmetries broken by the brane is the origin of the zero modes of the graviton on W^{p+1} that describe the brane degrees of freedom in the 'static' gauge (the statement in [11, 9, 30] that a p-brane does not carry any local degrees of freedom in the presence of dynamical gravity refers to the oscillating degrees of freedom -polarizations- of the graviton and gravitino, as discussed above, not to the zero modes). We thank W. Siegel for an illuminating discussion on this point.

supermembrane fermions one makes, following the above prescription, the local supersymmetry transformations (2.47)–(2.49) of the supergravity fields in (2.45) and identifies the (pull-back of the) parameter of these transformations with the supermembrane fermionic field, $\epsilon(\hat{x}(\xi)) = \hat{\theta}(\xi)$. The result at first order in $\hat{\theta}(\xi)$ is given by

$$* \, \hat{e}^a \wedge \hat{\psi}^\beta \, (\Gamma_a(1-\bar{\gamma}))_{\beta\alpha} + \delta_{\hat{\epsilon}=\hat{\theta}} \left(*\hat{e}^a \wedge \hat{\psi}^\beta \, (\Gamma_a(1-\bar{\gamma}))_{\beta\alpha} \right) = 0 \,, \tag{3.3}$$

or, in more detail ($\hat{e}^a_m = \partial_m \hat{x}^\mu e^a_\mu(\hat{x})$, $\hat{\psi}^\alpha_m = \partial_m \hat{x}^\mu \psi^\alpha_\mu(\hat{x})$, $\hat{\Gamma}^k = g^{km}(\xi) \hat{e}^a_m \Gamma_a$, $g_{mn}(\xi) = \hat{e}^a_m \hat{e}_{na}$),

$$\begin{aligned} \hat{*}\hat{e}^a \wedge \Bigg(& \hat{\psi}\Gamma_a(1-\bar{\gamma}) + \mathcal{D}\hat{\theta}\Gamma_a(1-\bar{\gamma}) + 2i\hat{\psi}_k \hat{\Gamma}^k \hat{\theta} \, \hat{\psi}\Gamma_a + \\ & + \frac{\epsilon^{mnk} \hat{e}^{b_1}_m \hat{e}^{b_2}_n}{\sqrt{|g(\xi)|}} (\hat{\psi}_k \Gamma^{b_3} \hat{\theta}) \, \hat{\psi}\Gamma_a \Gamma_{b_1 b_2 b_3} \Bigg)_\alpha + \\ & + 2i\hat{*}\hat{e}^b \wedge (\hat{\psi}\Gamma_a(1-\bar{\gamma}))_\alpha (\hat{\psi}^l \Gamma_b \hat{\theta} + \psi_b(\hat{x}) \hat{\Gamma}^l \hat{\theta}) \, \hat{e}_l{}^a - \\ & - 2i\hat{*}\hat{\psi}\Gamma^a \hat{\theta} \wedge (\hat{\psi}\Gamma_a(1-\bar{\gamma}))_\alpha = 0 \,, \end{aligned} \tag{3.4}$$

where again the generalized covariant derivative $\mathcal{D}$ is given by eqs. (2.40), (2.41) and, thus, includes a contribution from the fluxes F_{abcd}. We have checked explicitly that eq. (3.4) *formally* coincides with the first order equation that can be obtained within the standard 'background superfield' approach [4−6, 1] (without setting $\hat{\psi} = 0$ as in [4−6, 1]). By 'formally' we mean that in the equations obtained in the standard framework the graviton, the gravitino and the gauge field strength are, strictly speaking, solutions of the 'free' supergravity equations, while in our case such a restriction is absent and one can use, e.g., solutions of the interacting system of equations.

Eq. (3.4) is rather complicated. A simpler one results when in (3.4) the gravitino field is set equal to zero. This gives

$$\hat{\psi}^\alpha = 0 : \quad \hat{*}\hat{e}_a \wedge \mathcal{D}\hat{\theta}^\beta \, (\Gamma^a(1-\bar{\gamma}))_{\beta\alpha} = 0 \,, \tag{3.5}$$

or, equivalently,

$$\mathcal{D}\hat{\theta}^\beta \wedge (i \, \hat{*}\hat{e}_a \Gamma^a_{\beta\alpha} + \hat{\hat{\Gamma}}^{(2)}_{\beta\alpha}) = 0 \,.$$

Eq. (3.5) formally coincides with the M2-brane Dirac equation which is obtained in [4, 5, 1] within the on-shell background superfield approach, namely by expanding eq. (2.6) in $\hat{\theta}$ for $\psi = 0$. To see this explicitly, one may use the expression (2.40), (2.41) for the generalized covariant derivative $\mathcal{D}$ in (3.5) and the worldvolume tensor notation ($\hat{\Gamma}_n := \hat{e}^a_n \Gamma_a$, $\hat{\Gamma}^n := g^{nm}(\xi) \hat{e}^a_m \Gamma_a$ *etc.*) to arrive at

$$\begin{aligned} \left(D_n \hat{\theta} + \frac{i}{18} \hat{e}^a_n \left(F_{ab_1b_2b_3} \, \hat{\theta} \Gamma^{b_1b_2b_3} - \frac{1}{8} F^{b_1b_2b_3b_4} \, \hat{\theta} \Gamma_{ab_1b_2b_3b_4} \right) \right)^\beta \left(\hat{\Gamma}^n (1-\bar{\gamma}) \right)_{\beta\alpha} = 0 \,, \\ D_n \hat{\theta}^\alpha := \partial_n \hat{\theta}^\alpha(\xi) - \frac{1}{4} \hat{e}^c_n \omega^{ab}_c \, \hat{\theta}^\beta(\xi) \Gamma_{ab\beta}{}^\alpha \,. \end{aligned} \tag{3.6}$$

In this form the interaction of the supermembrane fermionic field with the A_3 'fluxes', this is to say with the field strength F_{abcd}, is manifest.

Linearizing eq. (3.4) in *all* the fermions, i.e. ignoring $\mathcal{O}(\hat{\theta}\,\hat{\psi})$, $\mathcal{O}(\hat{\psi}^{\wedge 2})$ together with the $\mathcal{O}(\hat{\theta}^{\wedge 2})$ contributions, we find the equation

$$\hat{*}\hat{e}^a \wedge (\mathcal{D}\hat{\theta} + \hat{\psi})\Gamma_a(1-\bar{\gamma}) = 0 \tag{3.7}$$

which includes the pull-back of the gravitino $\hat{\psi}$ and the Goldstone fermion $\hat{\theta}$ in the combination $(\mathcal{D}\hat{\theta}+\hat{\psi})$ only, which is invariant under the linearized supersymmetry. This observation supports the discussed fact (see footnote 6 and above) that, in the gauge $\hat{\theta} = 0$, the zero modes describing the brane fermionic degrees of freedom appear in the pull-back $\hat{\psi}$ of the bulk gravitino to W^3.

3.2 On the contribution of the supermembrane fermionic field to the full set of interacting equations

From the point of view of the interacting system, the setting $\hat{\psi} = 0$ above (and in [1, 2, 4−6]) or, taking into account the previous supersymmetry transformations that make manifest the Goldstone degrees of freedom, $\hat{\psi}^\alpha := \psi^\alpha(\hat{x}(\xi)) = \mathcal{D}\hat{\theta}^\alpha(\xi)$ is a kind of ansatz, or boundary condition, for the gravitino field on W^3. As such, its consistency with the supergravity equations should be checked. This is a convenient point to begin discussing the contribution of the supermembrane fermionic fields to the complete system of interacting equations, which includes the field equations whose gauge-fixed form is given by eqs. (2.34), (2.36) and (2.35).

It is natural to consider the above relation $\hat{\psi}^\alpha = \mathcal{D}\hat{\theta}^\alpha(\xi)$ on W^3 as produced by the ansatz

$$\psi^\alpha(x) = \mathcal{D}\tilde{\theta}^\alpha(x) \tag{3.8}$$

for the bulk gravitino, where the tilde denotes function on spacetime. Here the defining property of the Volkov-Akulov Goldstone fermion (see [22, 23]) $\tilde{\theta}^\alpha(x)$ is that its pull-back on W^3 coincides with the supermembrane fermionic field,

$$\tilde{\theta}^\alpha(\hat{x}(\xi)) = \hat{\theta}^\alpha(\xi)\,. \tag{3.9}$$

The irrelevance of the properties of $\tilde{\theta}^\alpha(x)$ outside the brane worldvolume W^3 is just the statement of the local supersymmetry of the 'free' supergravity action.

However, a direct substitution of the ansatz (3.8), (3.9) into the gravitino equations (2.35) would produce a problem. After some algebra (e.g. using identities from [19] and (2.42)) one finds that such a Volkov-Akulov Goldstone fermion $\tilde{\theta}^\alpha(x)$ would obey $i\tilde{\theta}^\beta(J_{10\,a}\Gamma^a_{\beta\alpha} + \frac{i}{2}J_8 \wedge \bar{\Gamma}^{(2)}_{\beta\alpha}) = 0$. This is equivalent (*cf.* section 2.3.2) to the condition $\hat{\theta}^\beta(\xi)(i\,\hat{*}\hat{e}^a\,\Gamma_{a\,\beta\alpha} + \frac{1}{2}\hat{e}^b \wedge \hat{e}^c\,\Gamma_{bc\,\beta\alpha}) = 0$ or, equivalently, $\hat{\theta}^\beta(\xi)(\hat{\Gamma}_n(1-\bar{\gamma}))_{\beta\alpha} = 0$ which implies the effective vanishing of the supermembrane fermionic field (actually $(1-\bar{\gamma})\hat{\theta} = 0$, but this in turn implies $\hat{\theta} = 0$, since the $(1+\bar{\gamma})\hat{\theta}$ part can be removed by the preserved supersymmetry gauge transformations which correspond to the κ-symmetry of the superbrane).

The reason for this apparent problem lies in the fact that the correct prescription to recover the supermembrane fermionic fields is to make the supersymmetry transformations of the gauge-fixed equations rather than using an ansatz like (3.8), (3.9) in them. Despite that the *r.h.s.* of (3.8) coincides with the gravitino supersymmetry transformations, its substitution into (2.35) does not automatically give the supersymmetry transformations of this equations. The point is that in a gauge-fixed equation where some Goldstone fields are set equal to zero, e.g. $\hat{\theta}^\alpha(\xi) = 0$, a zero in the *r.h.s.* of this equation may come from a term proportional to $\hat{\theta}^\alpha(\xi)$. As it is suggested by the study of the superfield description of the $D = 4$, $N = 1$ supergravity-superparticle and supergravity-superstring systems [8, 10], this is exactly the case for the gauge fixed form of the gravitino equation (2.35). Namely, the fully supersymmetric (not gauge-fixed) counterpart of this equation contains a *r.h.s.* proportional to $\hat{\theta}^\alpha(\xi)$. Schematically,

$$\Psi_{10\,\alpha} := \mathcal{D}\psi^\beta \wedge \bar{\Gamma}^{(8)}_{\beta\alpha} = \mathcal{O}(\hat{\theta}^\alpha(\xi))\,. \tag{3.10}$$

In other words, $\Psi_{10\,\alpha} \propto \hat{\theta}^\alpha$ rather than zero like in eq. (2.35) which comes from (3.10) in the gauge $\hat{\theta} = 0$. Then, taking into account the presence of a right hand side proportional to $\hat{\theta}^\alpha$ in a fully supersymmetric (not gauge-fixed) counterpart of eq. (2.35), one can use a local supersymmetry transformation to find an approximate expression for this *r.h.s.* ($\mathcal{O}(\hat{\theta})$ in (3.10)) up to the first order in $\hat{\theta}^\alpha$. This suggests a way of deriving the contributions of the supermembrane Goldstone fermion $\hat{\theta}^\alpha(\xi)$ to the supergravity equations from the local supersymmetry transformations of the gauge-fixed system of interacting equations (2.34), (2.35), (2.36) or of the gauge-fixed interacting action (2.27), which should work at least in low orders in $\hat{\theta}^\alpha(\xi)$.

As a first step in this direction let us derive the gravitino vertex operator of [3] and, thus, find the contribution proportional to $\hat{\theta}^\alpha$ in the right hand side of the fermionic field equation (3.10).

3.3 Gravitino vertex operator, a simple derivation of the 'fermionic equation for bosonic brane' and the Dirac action for the supermembrane fermionic field

The supersymmetry variation of the supergravity fields in the bosonic membrane action gives

$$\delta_\varepsilon S^0_{M2} = \frac{1}{2}\int_{W^3} \hat{*}\hat{e}_a \wedge \delta_\varepsilon e^a - \int_{W^3} \delta_\varepsilon \hat{A}_3 = -i\int_{W^3} \hat{*}\hat{e}_a \wedge \hat{\psi}^\beta\,(\Gamma^a(1-\bar{\gamma}))_{\beta\alpha}\,\varepsilon^\alpha(\hat{x})\,, \tag{3.11}$$

Notice that the requirement that this variation is zero for an arbitrary value of the parameter $\varepsilon^\alpha(\hat{x}(\xi))$, $\delta_\varepsilon S_{M2} = 0$ results in

$$\delta_\varepsilon S_{M2} = 0 \quad \forall \varepsilon^\alpha(\hat{x}) \qquad \Rightarrow \qquad \hat{*}\hat{e}_a \wedge \hat{\psi}^\beta\,(\Gamma^a(1-\bar{\gamma}))_{\beta\alpha} = 0\,, \tag{3.12}$$

which is exactly the 'fermionic equation for the bosonic brane', eq. (2.45). This easy way to derive eq. (2.45) is, actually, equivalent to a more involved derivation through the consistency conditions for the bulk field equations (see section 2.3.2 and [11]). As a byproduct one also easily sees that a supersymmetry transformation with parameter restricted by (2.31), $\hat{\varepsilon}^\alpha := \hat{\varepsilon}^\alpha(\hat{x}(\xi)) = (1+\bar{\gamma})^\alpha{}_\beta \kappa^\beta(\xi)$, leaves the action S^0_{M2} and hence $S_{SG} + S^0_{M2}$ invariant,

$$\delta_\varepsilon S^0_{M2}|_{\hat{\varepsilon}=(1+\bar{\gamma})\kappa(\xi)} = 0 \qquad \Rightarrow \qquad \delta_\varepsilon (S_{SG}[e^a, \psi^\alpha, A_3] + S_{M2}[\hat{e}^a, \hat{A}_3])|_{\hat{\varepsilon}=(1+\bar{\gamma})\kappa(\xi)} = 0\,, \tag{3.13}$$

which follows from the fact that $(1-\bar{\gamma})(1+\bar{\gamma}) = 0$. This preserved supersymmetry, coming from the κ-symmetry of the superbrane, allows one to extend the identification of broken supersymmetry with physical fermionic degrees of freedom of supermembrane, $(1-\bar{\gamma})\hat{\varepsilon} = (1-\bar{\gamma})\hat{\theta}$ (as in [21]), to a full identification of the pull-back to W^3 of supersymmetry parameter with the fermionic field,

$$\hat{\varepsilon}^\alpha = \varepsilon^\alpha(\hat{x}(\xi)) = \hat{\theta}^\alpha(\xi)\,. \tag{3.14}$$

With such an identification the expression for the interacting action becomes

$$\begin{aligned} S_{SG-M2} &= S_{SG}[e^a, \psi^\alpha, A_3] + S_{M2} \\ &= S_{SG}[e^a, \psi^\alpha, A_3] + S^0_{M2}[\hat{e}^a, \hat{A}_3] - i\int_{W^3} \Xi_{3\alpha}\,\hat{\theta}^\alpha + \mathcal{O}(\hat{\theta}^2)\,, \end{aligned} \tag{3.15}$$

where the first two terms in the *l.h.s.* describe the gauge-fixed action (2.27), while the third term (*cf.* (2.45)),

$$\int_{W^3} \Xi_{1\alpha}(\xi)\hat{\theta}^\alpha := i\delta_{\hat{\varepsilon}=\hat{\theta}} S_{M2}\,, \qquad \Xi_{3\alpha}(\xi) := \hat{*}\hat{e}_a \wedge \hat{\psi}^\beta\,(\Gamma^a(1-\bar{\gamma}))_{\beta\alpha}\,, \tag{3.16}$$

is given by the supersymmetry variation of S^0_{M2} by substituting $\hat{\theta}$ for $\hat{\varepsilon}$. This first order contribution determines the supermembrane fermionic vertex operator $\mathcal{V}$ as defined in [3],

$$i\delta_{\hat{\varepsilon}=\hat{\theta}} S_{M2} = \int_{W^3} \Xi_{3\alpha}(\xi)\hat{\theta}^\alpha = \int_{W^3} d^3\xi\, \psi_\mu{}^\beta(\hat{x})\, \mathcal{V}_\beta{}^\mu(\xi)\,, \tag{3.17}$$

$$d^3\xi\, \mathcal{V}_\beta{}^\mu(\xi) = d\hat{x}^\mu(\xi) \wedge \hat{*}\hat{e}_a \left(\Gamma^a(1-\bar{\gamma})\hat{\theta}(\xi)\right)_\beta = d\hat{x}^\mu \wedge \left(\hat{*}\hat{e}_a(\Gamma^a\hat{\theta})_\beta - i(\hat{\bar{\Gamma}}^{(2)}\hat{\theta})_\beta\right). \tag{3.18}$$

Thus, starting from a full but gauge-fixed description of the supergravity-superbrane interaction of [8, 9], we reproduce the supermembrane vertex operator from [3].

Notice that calculations as those above may apply equally well to the action (2.27) of the interacting system and to the action S_{M2} of a bosonic membrane in a spacetime supergravity background. Of course in the latter case the local supersymmetry is not a gauge transformation of the action but rather a transformation of the background fields.

By construction, the action (3.15) is invariant under full local supersymmetry (not just one-half as the gauge-fixed action (2.27)) up to contributions proportional to $\hat{\theta}$. Indeed, the Goldstone nature of $\hat{\theta}$ implies $\delta_\varepsilon \hat{\theta}(\xi) = -\varepsilon(\hat{x}) + \mathcal{O}(\hat{\theta})$ which, in the light of (3.17) and of the supersymmetry invariance of S_{SG}, gives $\delta_{\hat{\varepsilon}}(S_{SG} + S_{M2}) = \delta_{\hat{\varepsilon}}(S_{SG} + S^0_{M2} - i\int \Xi_3\hat{\theta}) = \mathcal{O}(\hat{\theta})$ for the action (3.15). To reach the supersymmetry invariance up to the first order in $\hat{\theta}$ one needs to recover the $\mathcal{O}(\hat{\theta}^{\wedge 2})$ components in the action. In our approach this can be done by adding $\frac{1}{2}\delta_{\hat{\varepsilon}=\hat{\theta}}(S_{SG} + S^0_{M2} - i\int \Xi_3\hat{\theta})$ to the action (3.15). This is just the term that should produce the supermembrane fermionic equation (3.4). One easily checks this for $\psi = 0$. Indeed,

$$\frac{1}{2}\delta_{\hat{\varepsilon}=\hat{\theta}}\left(S^0_{M2} - i\int \Xi_3\hat{\theta}\right)_{|\hat{\psi}=0} = -\frac{i}{2}\int \delta_{\hat{\varepsilon}=\hat{\theta}}\,(\Xi_3)_{|\hat{\psi}=0}\;\hat{\theta} = -\frac{i}{2}\int \hat{*}\hat{e}_a\wedge\mathcal{D}\hat{\theta}\Gamma^a(1-\bar{\gamma})\hat{\theta} \quad (3.19)$$

produces the Dirac equation (3.5).

The action (3.15), linear in $\hat{\theta}$, allows us to derive a supersymmetric set of interacting equations for the supergravity-supermembrane system with the same accuracy. For instance, the supersymmetric gravitino equation reads (*cf.* (2.35); notice that $\Gamma^a(1-\bar{\gamma})\hat{\theta} = \hat{\theta}(1+\bar{\gamma})\Gamma^a = \hat{\theta}\Gamma^a(1-\bar{\gamma})$)

$$\Psi_{10\,\alpha} := \mathcal{D}\psi^\beta\wedge\bar{\Gamma}^{(8)}_{\beta\alpha} = J_{10\,\alpha}[\hat{\theta}] + \mathcal{O}(\hat{\theta}^2)\,, \quad (3.20)$$

$$J_{10\,\alpha}[\hat{\theta}] = \frac{i}{2e(x)}e_b^{\wedge 10}\int_{W^3}\hat{*}\hat{e}_a\wedge\hat{e}^b\left(\hat{\theta}(\xi)\Gamma^a(1-\bar{\gamma})\right)_\alpha\delta^{11}(x-\hat{x}(\xi))\,. \quad (3.21)$$

Now, removing the bulk fermion by inserting the ansatz (3.8) for $\psi(x)$ in (3.20) and ignoring higher order terms in $\hat{\theta}$, one finds the relation between the bosonic currents (2.32), (2.33) and the fermionic current (3.21),

$$\tilde{\theta}(x)\left(iJ_{10\,a}\Gamma^a_{\beta\alpha} - \frac{1}{2}J_8\wedge\bar{\Gamma}^{(2)}_{\beta\alpha}\right) = J_{10\,\alpha}[\hat{\theta}]\,, \quad (3.22)$$

which is satisfied identically for a Goldstone fermion obeying (3.9). This shows that it is consistent to use the ansatz (3.8) to study particular solutions for the interacting system of supergravity and superbrane. Although this consistency is widely believed, the above is, to our knowledge, its first explicit check within the fully interacting system.

The study of the first order contribution in $\hat{\theta}$ to the full system of interacting equations for the D=11 supergravity-supermembrane system, as well as for systems including M5-brane and D=10 Dirichlet superbranes is a problem for further study. Another interesting question is whether one can extend the present approach to include contributions of higher order in $\hat{\theta}$ by using a counterpart of Noether method (see [24]) or, better still, the gauge completion procedure (see [13, 4]) but applied to the action as a whole rather than to the construction of the supervielbein and other separate superfields.

4. Conclusions and discussion

In this paper we have shown how the Dirac equation (3.5) [(3.4) for $\psi \neq 0$] for the fermionic coordinate field $\hat{\theta}(\xi)$ of the supermembrane (see [4−6, 1]) can be reproduced from a complete but gauge-fixed lagrangian description of the D=11 supergravity-supermembrane interacting system [8, 9]. This component spacetime lagrangian description is provided by the sum of the Cremmer-Julia-Scherk supergravity action [15] and a bosonic brane action given by the purely bosonic ($\hat{\theta} = 0$) 'limit' of the supermembrane action [12]. It preserves half of the local supersymmetry [11] reflecting the κ-symmetry of the superbrane action. From the point of view of the hypothetical superfield action for the supergravity-supermembrane interacting system the above spacetime description appears [8−10] as a result of fixing the superdiffeomorphism and superspace Lorentz symmetry by choosing the Wess-Zumino gauge for the supergravity superfields and of fixing (half of) the local supersymmetry by the $\hat{\theta}^\alpha(\xi) = 0$ gauge for the superbrane.

Formulated as a general prescription, our way of deriving the superbrane equations of motion consists in performing a spacetime local supersymmetry transformation [δ_ε of eqs. (2.47)–(2.49)] on the component fields that appear in the 'fermionic equation for bosonic brane' [11] [$\hat{\Xi}_{3\alpha} = 0$, eq. (2.45)], and then identifying the (pull-back of the) parameter of this transformation with the superbrane fermionic field $\hat{\theta}(\xi)$ [thus $\hat{\Xi}_{3\alpha} + \delta_{\hat{\varepsilon}=\hat{\theta}} \hat{\Xi}_{3\alpha} = 0$]. The identification of the $\hat{\theta}(\xi)$ with the parameter of the supersymmetry ($\hat{\varepsilon} = \hat{\theta}$) is made possible by the Goldstone nature of this superbrane fermionic field: its (non-pure gauge with respect to the κ-symmetry) components are the Goldstone fermions for the supersymmetries spontaneously broken by the superbrane [21].

The original 'fermionic equation for the bosonic brane' ($\hat{\Xi}_{3\alpha} = 0$, eq. (2.45)) is obtained as a consistency condition for the bosonic and fermionic field equations of the gauge fixed description of the supergravity-superbrane interacting system [8−10] which *does not* involve the superbrane fermionic $\hat{\theta}^\alpha(\xi)$ variable explicitly.[6] Here, in section 3.1, we have also shown how this 'fermionic equation for the bosonic brane' (2.45) can be obtained in an equivalent but very simple way, using as above the local supersymmetry transformation with $\hat{\varepsilon} = \hat{\theta}$, but for the bosonic brane action. In this way one also recovers the gravitino vertex operator of [3]. One may also notice that the 'fermionic equation for bosonic brane' formally coincides with the result of setting $\hat{\theta}^\alpha(\xi) = 0$ in the most general form of the superfield fermionic equations for superbranes in an on-shell superfield supergravity background, eqs. (2.6). Namely the leading component of (2.6) gives (2.45) but with the graviton and the gravitino satisfying the 'free' supergravity equations of motion, which is not the case for eq. (2.45) derived from the complete spacetime lagrangian description. Moreover, this situation holds at least at first order in $\hat{\theta}$ for the fermionic equations of motion and at second order in $\hat{\theta}$ for the action, namely our equation for the supermembrane Goldstone fermion $\hat{\theta}(\xi)$ also coincides (formally) with the equations derived in [4, 5, 1].

This shows, as widely believed, that the linearized equation for $\hat{\theta}(\xi)$ derived from the standard on-shell superfield approach to the supergravity background is still valid for the case of background fields that are not restricted by the 'free' supergravity equations, in spite of the fact that the on-shell constraints implying these 'free' supergravity equations were an essential ingredient in the derivation of the Dirac equation within the usual background on-shell superfield approach. Notice that our results also fit with those of [4] where it was found that, although the complete κ-symmetry of the supermembrane action (2.1) in curved superspace requires that the supervielbein $E_M^A(Z)$ and the super-3-form $A_3(Z)$ obey the on-shell supergravity constraints, the requirement of κ-symmetry up to the first

[6]Our approach makes particularly clear why the Dirac equation for the superbrane in a supergravity background with $\hat{\psi} = 0$ contains the same generalized covariant derivative ($\mathcal{D} = D - t = d - \omega - t$) involved in the gravitino supersymmetry transformation rules, a point also emphasized in section 3 of a recent paper [31], where our $\mathcal{D}\hat{\theta}$ is denoted by $\delta\psi\,\theta$. In the standard on-shell superfield approach such a coincidence can be traced to the fact that the component supersymmetry transformations may be deduced from the on-shell superspace constraints of [13, 14].

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order in $\hat{\theta}$ for the action written up to the second order in $\hat{\theta}$ does not impose any restrictions on the component background fields. Namely [4], if the on-shell supergravity constraints are used to decompose the action (2.1) in powers of $\hat{\theta}$ neglecting $\mathcal{O}(\hat{\theta}^{\wedge 3})$ terms and, then, the κ-symmetry is checked neglecting $\mathcal{O}(\hat{\theta}^{\wedge 2})$ terms, the result is that, surprisingly, such a weakened κ-symmetry requirement does not restrict the background fields of the supergravity multiplet by any equations of motion. An important question is whether this is also the case for the decomposition of the standard supermembrane action including higher order $\mathcal{O}(\hat{\theta}^{\wedge 3})$ terms in $\hat{\theta}$, and, if so, whether such a decomposition would coincide with the action obtained by a development of the approach of the present paper.

Within the on-shell background superfield approach such calculations, also technically involved, are possible using the recent results of [17]. To obtain equations of motion with higher order $\hat{\theta}(\xi)$ terms in present approach one has to perform a *'non-infinitesimal' supersymmetry transformation* up to some power in the parameter; the finite supersymmetry transformation, if found, might produce the fully supersymmetric (not gauge-fixed) action, if exists. For the existence of such finite transformation it is important that the local supersymmetry of the component gauge fixed description of the supergravity-supermembrane system is closed at least on shell. We have shown in section 2.2.4 that this is indeed the case and that this follows from the closure of the local supersymmetry of free supergravity.[7] A practical way to pursue the above proposed procedure method to find the action up to the terms of higher order in $\hat{\theta}(\xi)$ is to use a counterpart of the *gauge completion* method (see [13]), *but applied to the action itself.* Namely, one makes an 'infinitesimal' supersymmetry transformation in the action written up to $\mathcal{O}(\hat{\theta}^{\wedge k})$ and recovers the next order in $\hat{\theta}(\xi)$, $\mathcal{O}(\hat{\theta}^{\wedge (k+1)})$, by identifying $\hat{\varepsilon} = \hat{\theta}(\xi)$; then one tries making such an action supersymmetric up to order $\mathcal{O}(\hat{\theta}^{\wedge k})$ by modifying the supersymmetry transformation rules of the $\hat{x}^{\mu}(\xi)$ and $\hat{\theta}^{\alpha}(\xi)$.[8] Such a procedure would also answer the question of whether a fully supersymmetric (not gauge-fixed) interacting action $S_{SG}(e^a(x), \psi^{\alpha}(x), A_3(x)) + S_{M2}(e^a(\hat{x}), A_3(\hat{x}); \hat{\theta}(\xi), \psi^{\alpha}(\hat{x}))$, with $S_{M2}(\hat{e}^a, A_3(\hat{x}); 0, \psi^{\alpha}(\hat{x})) = S^0_{M2}(\hat{e}^a, A_3(\hat{x}))$, exists formulated only in terms of the physical fields of the supergravity multiplet and the superbrane Goldstonions $\hat{x}(\xi)$ and $\hat{\theta}(\xi)$. As we discussed in this paper (and may gathered from the results of [4]), the answer to this question is affirmative up to second order in $\hat{\theta}(\xi)$. Notice that, if an obstruction were found at some higher order in $\hat{\theta}$, it would pose an interesting dilemma: whether such an obstruction is the result of a non-lagrangian nature of the equations of motion for the physical fields of the supergravity multiplet in the interacting system, or whether it is the application of the above procedure to the equations of motion for the physical fields of the supergravity multiplet that fails. The second alternative would imply the impossibility of finding a fully supersymmetric

[7]It would be interesting to study the algebra of the spacetime local supersymmetry of the $D = 11$ supergravity interacting with M5-brane and of the $D = 10$ supergravity interacting with higher Dirichlet branes. The (spacetime, gauge-fixed) lagrangian description of such interactions implies the use of the duality-invariant formulations of supergravity (see [25] for $D = 11$, [26] for $D = 10$ type-IIA and [27] for $D = 10$ type IIB) where the commutator of two supersymmetry transformations leaving invariant the supergravity action would involve the PST (Pasti-Sorokin-Tonin) gauge transformations.

[8]To lowest order $\delta_{\varepsilon}\hat{x}^{\mu}(\xi) = -i\hat{\theta}\Gamma^a \varepsilon(\hat{x})\, e_a{}^{\mu}(\hat{x}) + \mathcal{O}(\hat{\theta})$, $\delta_{\varepsilon}\hat{\theta}^{\alpha}(\xi) = -\varepsilon^{\alpha}(\hat{x}(\xi)) + \mathcal{O}(\hat{\theta})$.

system of equations for the physical fields of the supergravity multiplet and the superbrane Goldstone fields. Although at first glance this would look discouraging, it might also point towards some hidden ingredients of M-theory.

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References

[1] R. Kallosh and D. Sorokin, *Dirac action on M5 and M2 branes with bulk fluxes*, *JHEP* **05** (2005) 005 [hep-th/0501081].

[2] P.K. Tripathy and S.P. Trivedi, *D3 brane action and fermion zero modes in presence of background flux*, *JHEP* **06** (2005) 066 [hep-th/0503072];
R. Kallosh, A.-K. Kashani-Poor and A. Tomasiello, *Counting fermionic zero modes on M5 with fluxes*, *JHEP* **06** (2005) 069 [hep-th/0503138];
L. Martucci, J. Rosseel, D. Van den Bleeken and A. Van Proeyen, *Dirac actions for D-branes on backgrounds with fluxes*, *Class. and Quant. Grav.* **22** (2005) 2745 [hep-th/0504041].

[3] K. Becker, M. Becker and A. Strominger, *Five-branes, membranes and nonperturbative string theory*, *Nucl. Phys.* **B 456** (1995) 130 [hep-th/9507158];
K. Becker and M. Becker, *Boundaries in M-theory*, *Nucl. Phys.* **B 472** (1996) 221 [hep-th/9602071].

[4] B. de Wit, K. Peeters and J. Plefka, *Superspace geometry for supermembrane backgrounds*, *Nucl. Phys.* **B 532** (1998) 99 [hep-th/9803209].

[5] J.A. Harvey and G.W. Moore, *Superpotentials and membrane instantons*, hep-th/9907026.

[6] M. Graña, *D3-brane action in a supergravity background: the fermionic story*, *Phys. Rev.* **D 66** (2002) 045014 [hep-th/0202118];
L. Martucci and P.J. Silva, *On type-II superstrings in bosonic backgrounds and their t-duality relation*, *JHEP* **04** (2003) 004 [hep-th/0303102];
D. Marolf, L. Martucci and P.J. Silva, *Fermions, T-duality and effective actions for D-branes in bosonic backgrounds*, *JHEP* **04** (2003) 051 [hep-th/0303209]; *Actions and fermionic symmetries for D-branes in bosonic backgrounds*, *JHEP* **07** (2003) 019 [hep-th/0306066]; *The explicit form of the effective action for F1 and D-branes*, *Class. Quant. Grav.* **21** (2004) S1385–S1390 [hep-th/0404197].

[7] I.A. Bandos, D.P. Sorokin, M. Tonin, P. Pasti and D.V. Volkov, *Superstrings and supermembranes in the doubly supersymmetric geometrical approach*, *Nucl. Phys.* **B 446** (1995) 79 [hep-th/9501113];
P.S. Howe and E. Sezgin, *Superbranes*, *Phys. Lett.* **B 390** (1997) 133 [hep-th/9607227]; *$D = 11$, $p = 5$*, *Phys. Lett.* **B 394** (1997) 62 [hep-th/9611008];

P.S. Howe, E. Sezgin and P.C. West, *Covariant field equations of the M-theory five-brane*, *Phys. Lett.* **B 399** (1997) 49 [hep-th/9702008];
D. Sorokin, *Superbranes and superembeddings*, *Phys. Rept.* **329** (2000) 1 [hep-th/9906142].

[8] I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo and J. Lukierski, $D = 4$ *supergravity dynamically coupled to a massless superparticle in a superfield lagrangian approach*, *Phys. Rev.* **D 67** (2003) 065003 [hep-th/0207139].

[9] I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo and J. Lukierski, *On dynamical supergravity interacting with super-p-brane sources*, in procs. *The 3rd International Sakharov Conference, Moscow, June 24-29, 2002*, Sci. World Pub. Co, A. Semikhatov et al. eds., vol. 2, pp. 312–324, 2003 [hep-th/0211065];
I.A. Bandos, *Supergravity interacting with superbranes and spacetime Higgs effect in general relativity*, *Symmetries in gravity and field theory*, Salamanca, June 9-11, 2003,, J.M. Cerveró and Pilar García–Estévez eds., Ediciones Univ. de Salamanca, **62**, pp. 489–504, 2004 [hep-th/0312095].

[10] I.A. Bandos and J.M. Isidro, $D = 4$ *supergravity dynamically coupled to superstring in a superfield lagrangian approach*, *Phys. Rev.* **D 69** (2004) 085009 [hep-th/0308102].

[11] I.A. Bandos, J.A. De Azcarraga and J.M. Izquierdo, *Supergravity interacting with bosonic p-branes and local supersymmetry*, *Phys. Rev.* **D 65** (2002) 105010 [hep-th/0112207].

[12] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Supermembranes and eleven-dimensional supergravity*, *Phys. Lett.* **B 189** (1987) 75; *Properties of the eleven-dimentional supermembrane theory*, *Ann. Phys. (NY)* **185** (1988) 330.

[13] E. Cremmer and S. Ferrara, *Formulation of eleven-dimensional supergravity in superspace*, *Phys. Lett.* **B 91** (1980) 61.

[14] L. Brink and P.S. Howe, *Eleven-dimensional supergravity on the mass - shell in superspace*, *Phys. Lett.* **B 91** (1980) 384.

[15] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409.

[16] J.A. de Azcarraga and J. Lukierski, *Supersymmetric particles with internal symmetries and central charges*, *Phys. Lett.* **B 113** (1982) 170;
W. Siegel, *Hidden local supersymmetry in the supersymmetric particle action*, *Phys. Lett.* **B 128** (1983) 397.

[17] D. Tsimpis, *Curved 11D supergeometry*, *JHEP* **11** (2004) 087 [hep-th/0407244].

[18] B. Julia and S. Silva, *On first order formulations of supergravities*, *JHEP* **01** (2000) 026 [hep-th/9911035].

[19] I.A. Bandos, J.A. de Azcarraga, M. Picon and O. Varela, *Generalized curvature and the equations of* $D = 11$ *supergravity*, *Phys. Lett.* **B 615** (2005) 127 [hep-th/0501007].

[20] M.J. Duff, R.R. Khuri and J.X. Lu, *String solitons*, *Phys. Rept.* **259** (1995) 213 [hep-th/9412184];
K.S. Stelle, *BPS branes in supergravity*, hep-th/9803116.

[21] J. Hughes, J. Liu and J. Polchinski, *Supermembranes*, *Phys. Lett.* **B 180** (1986) 370;
J. Hughes and J. Polchinski, *Partially broken global supersymmetry and the superstring*, *Nucl. Phys.* **B 278** (1986) 147.

[22] D.V. Volkov and V.P. Akulov, *Is the neutrino a goldstone particle?*, *Phys. Lett.* **B 46** (1973) 109.

[23] D.V. Volkov and V.A. Soroka, *Higgs effect for Goldstone particles with spin 1/2*, *JETP Lett.* **18** (1973) 312.

[24] P. van Nieuwenhuizen, *Supergravity*, *Phys. Rept.* **68** (1981) 189.

[25] I.A. Bandos, N. Berkovits and D.P. Sorokin, *Duality-symmetric eleven-dimensional supergravity and its coupling to M-branes*, *Nucl. Phys.* **B 522** (1998) 214 [hep-th/9711055].

[26] I.A. Bandos, A.J. Nurmagambetov and D.P. Sorokin, *Various faces of type-IIA supergravity*, *Nucl. Phys.* **B 676** (2004) 189 [hep-th/0307153].

[27] G. Dall'Agata, K. Lechner and D.P. Sorokin, *Covariant actions for the bosonic sector of* $D = 10$ *IIB supergravity*, *Class. and Quant. Grav.* **14** (1997) L195 [hep-th/9707044];
G. Dall'Agata, K. Lechner and M. Tonin, $D = 10$*,* $N = IIB$ *supergravity: Lorentz-invariant actions and duality*, *JHEP* **07** (1998) 017 [hep-th/9806140].

[28] R. Altendorfer, J. Bagger and D. Nemeschansky, *Supersymmetric Randall-Sundrum scenario*, *Phys. Rev.* **D 63** (2001) 125025 [hep-th/0003117];
J. Bagger and D. Belyaev, *Twisting warped supergravity*, *JHEP* **06** (2003) 013 [hep-th/0306063]; *Brane-localized goldstone fermions in bulk supergravity*, hep-th/0406126 and refs. therein.

[29] T. Adawi, M. Cederwall, U. Gran, B.E.W. Nilsson and B. Razaznejad, *Goldstone tensor modes*, *JHEP* **02** (1999) 001 [hep-th/9811145];
M. Cederwall, U. Gran, M. Holm and B.E.W. Nilsson, *Finite tensor deformations of supergravity solitons*, *JHEP* **02** (1999) 003 [hep-th/9812144].

[30] I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo and J. Lukierski, *Gravity, p-branes and a spacetime counterpart of the Higgs effect*, *Phys. Rev.* **D 68** (2003) 046004 [hep-th/0301255].

[31] E. Bergshoeff, R. Kallosh, A.-K. Kashani-Poor, D. Sorokin and A. Tomasiello, *An index for the dirac operator on D3 branes with background fluxes*, hep-th/0507069.